SIMPLE FRAME PATTERN OCCURRENCES IN CYCLES

MILES JONES (JOINT WORK WITH SERGEY KITAEV AND JEFF REMMEL)

I will define the notion of a *frame pattern* occurrence in a cycle. In particular I will look at the simplest of frame patterns that I will call μ. There are a few interesting integer sequences that come from the number of cycles that have k μ-occurrences.

Let us denote the number of n-cycles that have exactly k *non-trivial* occurrences of μ, $NT_{n,k}(\mu)$. We have shown that these numbers are given by $NT_{n,k}(\mu) = \sum_{i=1}^{2k+1} c_i \binom{n-1}{i}$ for certain constants c_i.

We have shown that the numbers $NT_{n,k}(\mu)$ for $k = \binom{n-1}{2}, \binom{n-1}{2} - 1, \ldots, \binom{n-2}{2}$ are the same as the number of ways to partition an integer $n = 1, 2, \ldots$. This is proven by plotting the non-occurrences of μ on a grid to form a Ferrer's board.

If a cycle $C = (c_0, \ldots, c_{n-1})$ has the property that $c_i + 1 \neq c_{i+1}$ then we call this cycle *incontractible*. The number of incontractible $(2n+1)$-cycles with the minimal number of μ occurrences is the nth Catalan number $\binom{2n}{n}\frac{1}{n+1}$. This is proven by looking at the *charge graph* which is a graph based on the Mahonian statistic *charge*. It turns out that an incontractible $(2n+1)$-cycle has the minimal number of μ occurrences if and only if its charge graph resembles a *Dyck path*.