ON MINIMAL RATIONAL ELLIPTIC SURFACES

ANTONIO LAFACE

Universidad de Talca
8 de mayo de 2014

A smooth complex projective rational surface X equipped with an elliptic fibration $\pi\colon X \to \mathbb{P}^1$ is minimal if no prime component of a fiber is a (-1)-curve. It is known since [1, 5] that the Cox ring [3] $\mathcal{R}(X)$ of X is finitely generated if and only if the classes of the prime components of the reducible fibers of π span a sublattice of $\mathrm{Pic}(X)$ of corank one. In this case a generating set for $\mathcal{R}(X)$ is known [2] when the fibration π admits a section $\sigma\colon \mathbb{P}^1 \to X$, that is $\pi \circ \sigma = \mathrm{id}$.

In general, that is without assuming that π admits a section, even the number of negative curves of X is not known. In this talk I intend to discuss a theorem about the number of (-1)-curves of X, showing that it equals the number of integer points of the Riemann-Roch polyhedron of a certain divisor of a toric variety canonically attached to X.

This is joint work in progress [4] with Damiano Testa.

References

[1] M. Artebani, A. Laface: *Cox rings of surfaces and the anticanonical Iitaka dimension*, Adv. Math. 226 (2011), no. 6, 5252–5267.
[2] M. Artebani, A. Garbagnati, A. Laface: *Cox rings of extremal rational elliptic surfaces*, to appear in Transactions of the AMS.
[3] I. Arzhantsev, U. Derenthal, J. Hausen, A. Laface: *Cox rings*, to appear in Cambridge studies in advanced mathematics.
[4] A. Laface, D. Testa: *On minimal rational elliptic surfaces*, in preparation.
[5] B. Totaro: *The cone conjecture for Calabi-Yau pairs in dimension 2*, Duke Math. J. 154 (2010), no. 2, 241–263.

Universidad de Concepción, email: antonio.laface@gmail.com.