Quadratic Forms in Chile 2018

Conference at Talca, 8–12 January 2018

Abstracts

Arenas Carmona, Luis: Geometry of the Bruhat-Tits tree – branches of orders and field extensions

Universidad de Chile (Chile)

Abstract

Let A be a quaternion algebra over a number field K. If Eichler's condition is satisfied, the theory of spinor class fields and representation fields allows us to describe the set of isomorphism classes of orders R of maximal rank in A, in a fixed genus, yielding an embedding of a fixed sub-order H.
Computing a representation field is equivalent to computing the relative spinor image. When E is an intersection of maximal orders, these computation can be performed through the geometry of the Bruhat-Tits tree, which can be thought as a p-adic analog of the uper half plane.
In a paper of 2013 we showed that this tree, whose vertices correspond to maximal orders in the local split quaternion algebra, containing a given suborder falls in a rather restricted family. In fact, through extensions of the field of scalars, we are restricted to study to essentially different shapes for these orders.

Aubert, Anne-Marie: On the Langlands-correspondence for classical p-adic groups

Université Paris VI (France)

Abstract

Let F be a non-archimedean local field, and G be the F-points of a connected reductive algebraic group defined over F. We will explain how one can conjecturally reduce the study of the Langlands correspondence for G to the knowledge of the correspondence for supercuspidal representations of the Levi subgroups of G.
We will illustrate our constructions in the case when G is a classical group, in particular a special orthogonal group of odd dimension.

Becker, Eberhard: On the Waring Problems for fields

TU Dortmund (Germany)

Abstract

The talk will focus on the case of formally real fields where we obtain bounds for the higher Pythagoras numbers in terms of the quadratic Pythagoras number.

Bolaños, Wilmar: On classification of cyclic quadratic spaces $\left\langle O_K, Tr_{K/Q}(x^2)\right\rangle$

Universidad de los Andes (Columbia)

Abstract

Classification of number fields $K/\mathbb{Q}$ and its respective maximal orders $\mathfrak{o}_K$ are an important topic of investigation. If K is a number field then its underlying quadratic module is the pair

$$\left\langle \mathfrak{o}_K, Tr_{K/\mathbb{Q}}(x^2)\right\rangle .$$

In general, the structure of this quadratic modules is unknown; however, when K is a cyclic number field of degree q, a prime number, we have the following results:

Theorem 1 (P.E. Conner, R. Perlis,1984) *Let F and K be cyclic number fields of degree q a odd prime such that $\mathfrak{d}(F) = \mathfrak{d}(E)$, where $\mathfrak{d}(K)$ denotes the discriminant of K. Then for any group isomorphism $h : Gal(F/Q) \to Gal(K/Q)$, then there exist an isomorphism*

$$\left\langle \mathfrak{o}_F, Tr_{F/\mathbb{Q}}(x^2)\right\rangle \simeq \left\langle \mathfrak{o}_K, Tr_{K/\mathbb{Q}}(x^2)\right\rangle .$$

Theorem 2 (G. Mantilla, 2010): *Let K be a cubic number field of positive fundamental discriminant. Let F be a number field such that there exist and isomorphism of quadratic modules*

$$\left\langle \mathfrak{o}_F^0, Tr_{F/\mathbb{Q}}(x^2)\Big|_{\mathfrak{o}_F^0}\right\rangle \simeq \left\langle \mathfrak{o}_K^0, Tr_{K/\mathbb{Q}}(x^2)\Big|_{\mathfrak{o}_K^0}\right\rangle$$

and assume $9 \nmid \mathfrak{d}(L)$. Then $K = L$.

Theorems 1 and 2 show us that under certain non-trivial conditions over the trace operator or the structure of the number fields K and L we can prove that their underlying quadratic modules belong to the same isomorphic class. The purpose of this talk is to exhibit a generalization of the theorem 1 for cyclic number field, specifically for cyclic number fields of degree q^ℓ, and to discuss possible generalizations of these theorems and its applications.

Chan, Billy: Waring's problem for integral quadratic forms

Wesleyan University (USA)

Abstract

For every positive integer n, let $g(n)$ be the smallest integer such that if an integral quadratic form in n variables can be written as a sum of squares of integral linear forms, then it can be written as a sum of $g(n)$ squares of integral linear forms. As a generalization of Lagrange's Four-Square Theorem, Mordell (1930) showed that $g(n) = n + 3$ when $n \leq 5$. More than sixty years later, M.-H. Kim and B.-K. Oh showed that $g(6) = 10$ and the growth of $g(n)$ is at most an exponential of n. In this talk, I will discuss a recent improvement of Kim and Oh's result showing that the growth of $g(n)$ is at most an exponential of $\sqrt{n}$. This is a joint work with Constantin Beli, Maria Icaza, and Jingbo Liu.

Chapman, Adam: The μ-Invariant and the Symbol Length in Kato-Milne Cohomology

Tel-Hai College (Israel)

Abstract

For a field of characteristic 2 of given finite μ-invariant, we provide explicit upper bounds for the symbol lengths of all Kato-Milne cohomology groups. In particular, we obtain that the symbol lengths are finite, giving a characteristic 2 analogue to a result previously obtained by Saltman and Krashen.

We also discuss the implications of having symbol length 1 in a certain cohomology group, generalizing among other things a result of Baeza's that the μ-invariant is 4 if and only if every two quaternion algebras share an inseparable quadratic splitting field. The talk is based on a joint work with Kelly McKinnie.

Colliot-Thélène, Jean-Louis: Rationality in families

Université Paris XI (France)

Abstract

Over the last three years, many works have been devoted to this topic. A central tool is the specialisation of Chow groups or of R-equivalence. The talk will be a survey of some of these many works. I plan to detail one of them, where the by now usual approach via explicit desingularisation may be avoided.

First, Uriya: Brauer classes supporting an involution

University of Haifa (Israel)

Abstract

Let K/F be a quadratic Galois field extension and let s be the nontrivial F-automorphism of K. A celebrated theorem of Albert characterizes the kernel of the corestriction map $Br(K) \to Br(F)$ as those Brauer classes containing a central simple K-algebra that admits an s-involution, i.e. an involution whose restriction to K is s. Saltman generalized this result from quadratic Galois extensions of fields to quadratic Galois extension of commutative rings. A later proof given by Knus, Parimala and Srinivas applies in the greater generality of unramified double covers of schemes. I will discuss a recent work with B. Williams in which we extend the aforementioned results to ramified double covers of schemes (and more generally, of locally ringed topoi). We further give a criterion to detect whether two s-involutions are locally isomorphic relative to the etale site of the base. Some new phenomena that can occur only in the ramified case will discussed as well.

Gładki, Paweł: Witt rings of quadratically presentable fields

University of Silesia (Poland)

Abstract

We introduce an approach to the axiomatic theory of quadratic forms based on presentable partially ordered sets, that is partially ordered sets subject to additional conditions which amount to a strong form of local presentability. It turns out that the classical notion of the Witt ring of symmetric bilinear forms over a field makes sense in the context of quadratically presentable fields, that is

fields equipped with a presentable partial order inequationaly compatible with the algebraic operations. In particular, Witt rings of symmetric bilinear forms over fields of arbitrary characteristics are isomorphic to Witt rings of suitably built quadratically presentable fields.

Gupta, Parul: Strong linkage for function fields

Universiteit Antwerpen (Belgium) / Technische Universität Dresden (Germany)

Abstract

For a field we study the property, called *strong linkage*, that any finite number of quaternion algebras have a common slot. The study of this field property is motivated by its relation to quadratic forms and further by the example of global fields. Using geometric techniques, we show that strong linkage holds for the following two types of fields:

(i) function fields of curves over a complete discretely valued field with algebraically closed residue field (e.g. $\mathbb{C}(\!(X)\!)(Y)$),

(ii) fraction fields of two-dimensional complete local domains with algebraically closed residue field (e.g. $\mathbb{C}(\!(X,Y)\!)$).

We further relate strong linkage to local-global principles for isotropy of quadratic forms for these type of fields.

Herrero, Sebastián: A p-adic Linnik problem with applications

Chalmers University of Technology / University of Gothenburg (Sweden)

Abstract

In the first part of this talk I will recall the classical Linnik problem about the distribution of lattice points on an euclidean sphere. These problems were introduced by Linnik in the 60's and can be solved by using the theory of modular forms of half-integral weight and bounds for their Fourier coefficients (proved by Iwaniec in 1967).

In the second part of this talk I will present a certain p-adic analogue of this problem which arises naturally in the study of the distribution of isogenies of growing degree in the ring of endomorphisms of a supersingular elliptic curve. This problem can also be solved by using the theory of modular forms.

This is joint work with Ricardo Menares (Pontificia U. Católica de Valparaíso) and Juan Rivera-Letelier (U. of Rochester).

Jacob, Bill: p^m-extensions in characteristic p

University of California Santa Barbara (USA)

Abstract

In earlier work the Brauer kernel was determined for extensions of degree p in characteristic $p > 2$ where the Galois group is a semidirect product of order ps for $s|(p-1)$. This result is extended here and tools are developed to compute the cohomological kernels $H^{n+1}_{p^m}(E_m/F)$ for all $n \geq 0$ where $[E_m : F] = p^m$ and the Galois closure is a semidirect product of cyclic groups order p^m and s where $s|(p-1)$. As an application it is shown that any F-division p-algebra of index p^m split in E_m is cyclic; a characteristic p analogue of a result of Vishne. This work is joint with Roberto Aravire and Manuel O'Ryan.

Kala, Vítězslav: Universal quadratic forms over number fields

Univerzita Karlova (Czech Republic)

Abstract

I will talk about several recent results concerning universal quadratic forms over real quadratic number fields (i.e., totally positive quadratic forms that represent all totally positive integers). Together with Valentin Blomer we have proved estimates on the minimal number of variables of a universal (diagonal) form, and with Pavlo Yatsyna we have studied the (non)existence of universal forms whose coefficients are rational integers. This partly relies on a description of the additive semigroup of totally positive integers (in terms of continued fraction coefficients), obtained with Tomas Hejda.

Karpenko, Nikita: On generic flag varieties of $Spin(11)$

University of Alberta (Canada)

Abstract

Let X be the variety of Borel subgroups of a split semisimple algebraic group G over a field, twisted by a generic G-torsor. Conjecturally, the canonical epimorphism of the Chow ring $CH(X)$ onto the associated graded ring $GK(X)$ of the topological filtration on the Grothendieck ring $K(X)$ is an isomorphism. We prove new cases $G = Spin(11)$ and $G = Spin(12)$ of this conjecture. On an equivalent note, we compute the Chow ring $CH(Y)$ of the highest orthogonal

grassmannian Y for the generic 11- and 12-dimensional quadratic forms of trivial discriminant and Clifford invariant. In particular, we describe the torsion subgroup of the Chow group $CH(Y)$ and determine its order which is equal to 16777216. On the other hand, we show that the Chow group of 0-cycles on Y is torsion-free.

Krashen, Daniel: Brauer classes on p-adic surfaces

University of Georgia (USA)

Abstract

In this talk, I will consider Brauer classes on p-adic function field, particularly focusing on new results on the period-index problem in the case of p-adic surfaces. Brauer classes control the complexity of many other algebraic structures, and thereby find applications in a variety of contexts. While various techniques have been developed which have informed our understanding of Brauer classes on p-adic curves and function fields of more general arithmetic surfaces, relatively little progress has been made in the case of higher dimensional schemes. In this talk I will discuss recent joint work with Antieau, Auel, Ingalls and Lieblich where we give explicit bounds for the index of Brauer classes over such fields.

Lucchini Arteche, Giancarlo: A reduction theorem for homogeneous spaces.

Universidad de Chile (Chile)

Abstract

Certain arithmetic properties of varieties, like the density of rational points in local points (eventually modulo a Brauer-Manin obstruction), have nice (meta-) properties such as being invariant by birational equivalence in certain families of varieties. In the context of homogeneous spaces of linear connected groups, one can use this kind of functoriality properties to reduce the study of weak approximation to the particular case of homogeneous spaces of SL_n with finite stabilizers. The goal of this talk is to explicitly give 3 functoriality conditions with which one can obtain such a reduction result in all generality. This raises the question of finding other nice properties (not necessarily of arithmetic order) one would expect to have in homogeneous spaces and that verify these 3 conditions, so that an eventual proof can be reduced to the case of finite stabilizers. I will finally try to explain the main ideas that go into the reduction result.

Leep, David: Systems of quadratic forms over complete discretely valued fields

University of Kentucky (USA)

Abstract

The main theorem states that a system of r quadratic forms in n variables defined over an arbitrary p-adic field K has a nontrivial zero over K as long as $n > 4r$. This bound is optimal. This result improves a theorem of Heath-Brown and answers a question first posed over 50 years ago by Birch and Lewis.

The theorem is obtained from a much more general result dealing with systems of quadratic forms defined over a complete discretely valued field K with residue field k. Suppose there is a constant A such that for each r, every system of r quadratic forms defined over k in more than Ar variables has a nontrivial zero over k. Then every system of r quadratic forms defined over K in more than $2Ar$ variables has a nontrivial zero over K. The theorem on systems of quadratic forms defined over a p-adic field follows as a corollary with $A = 2$ by a theorem of Chevalley applied to systems of quadratic forms defined over finite fields.

As is well known, this theorem has important applications to computing u-invariants of arbitrary function fields over p-adic fields and over other complete discretely valued fields.

Mantilla Soler, Guillermo: On the arithmetic determination of the trace

Universidad de los Andes (Colombia)

Abstract

One of the most basic arithmetic invariants of a number field K is its discriminant. Even though the discriminant is not a complete invariant, at least for degree bigger than 2, the problem of classifying number fields by means of their discriminants is central in algebraic number theory. Since the discriminant of a number field is the determinant associated the trace pairing, it is particularly interesting to ask when for two number fields such pairings are equivalent. One of our latest results on the classification of the trace is that for number fields ramified at infinity, with only tame ramification, the integral trace pairing is totally determined by the discriminant, the signature, and a finite set of positive integers that depend only on the factorization of ramified primes. In particular, for non-totally real number fields we see that in the absence of wild rami-fication, the integral trace form is determined by the arithmetic of the number field at ramified primes.

Merkurjev, Alexander: Rationality problem for classifying spaces of spinor groups

University of California Los Angeles (USA)

Abstract

The classifying space $BSpin_n$ of the spinor group $Spin_n$ parameterizes quadratic forms of dimension n, trivial discriminant and trivial Clifford invariant. The retract rationality property of $BSpin_n$ amounts to *parametrization* of such forms by algebraically independent parameters. The space $BSpin_n$ is known to be retract rational for $n \leq 14$. In the talk, I will discuss retract rationality property of $BSpin_n$ for all n.

Parimala, Raman: Cohomological invariants of G-Galois algebras

Emory University (USA)

Abstract

Let G be a finite group. We explain the construction of degree two cohomological invariants for G-Galois algberas. These invariants provide criterion for the existence of self-dual normal basis for G-Galois algebras for special classes of fields and for special classes of groups. (joint with E. Bayer-Fluckiger)

Rossetti, Juan Pablo: Positive definite quadratic forms with the same theta series

Universidad Nacional de Cordoba (Argentina)

Abstract

Given a positive definite quadratic form, we associate to it a Euclidean lattice. Two lattices are called "isospectral" when the quadratic forms have the same theta series.

There are some examples of isospectral lattices, started in dimension 16, reduced to 12, 8, 6, 5 and 4. Schiemann showed that in dimension 3 there cannot be examples.

All known examples of isospectral lattices satisfy the condition that you can divide the lattices into a finite number or pieces, and there is a bijection between the pieces in one lattice and those in the other one, which preserves lengths of vectors, moreover, the corresponding pieces are isometric. The "jigsaw conjecture" claims that all isospectralities must satisfy this condition.
In this talk we will present an elementary construction of a pair of isospectral (non isometric) lattices in dimension 5, which can be divided into only two parts each, and the first parts are the same in both lattices, and the second ones are isometric.
On the other hand, we propose to study isospectrality of lattices with respect to the norm 1. The motivation is that lens spaces turn out to be isospectral when certain lattices naturally associated to them are isospectral with respect to the norn one. This has been shown in a work in collaboration with Emilio Lauret and Roberto Miatello. The subject has not been intensively studied yet.

Tignol, Jean-Pierre: Arason's filtration of the Witt group of dyadic fields (after Verstraete)

Université Catholique de Louvain (Belgium)

Abstract

Arason recently defined a filtration of the Witt group of quadratic forms over a discretely valued field of residue characteristic 2:

$$W_0 \subset W_1 \subset W_2 \subset \cdots \subset W_n \subset \cdots \subset W_q(F)$$

in which W_0 is the group of quadratic forms split by the maximal tame extension of F. This talk will report on work by Joachim Verstraete in which the quotients W_n/W_{n-1} are given an interpretation as Witt groups built from totally singular quadratic forms over the residue field.

Urzúa, Giancarlo: Some ways to compute the fundamental group of a Godeaux surface

Pontificia Universidad Católica (Chile)

Abstract

Miles Reid conjectured in the '70s that the moduli space of Godeaux surfaces consists of five unirational 8 dimensional irreducible components, each of which labelled by the five possible étale fundamental groups $\{1\}, Z/2, Z/3, Z/4, Z/5$. Reid himself proved it for $Z/3, Z/4$ and $Z/5$. Moreover it is believed that for Godeaux surfaces the étale and topological fundamental group coincide. Reid verified it for $Z/4$ and $Z/5$. In a joint work with S. Coughlan (2016) we verified it for $Z/3$ using Reid's moduli space. For $\{1\}$ and $Z/2$ the conjecture is still open, and we only know examples of such surfaces. Recently (2015) in a joint work with J. Rana and J. Tevelev we showed that a certain étale trivial Godeaux surface (so called Craighero-Gattazzo surface) was indeed simply connected. In this talk I will show the techniques used in these computations, which involve topology, deformation theory and algebraic reduction mod p.

Vishik, Alexander: Motives of affine quadrics

University of Nottingham (Great Britain)

Abstract

Affine quadrics can be considered as non-split spheres. It appears that their motives behave much better than that of projective quadrics. In particular, the motive of an affine quadric $\{q = 1\}$ determines the respective quadratic form q.