



Coloquio Inst-Mat

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Combinatorics, Categorification, and Crystals

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Abstract

Categorification attempts to replace algebraic and geometric structures with more general categories. It has enjoyed amazing successes, such as Khovanov homology categorifying the Jones polynomial, KLR algebras categorifying quantum groups, or Soergel bimodules categorifying Hecke algebras. The payoffs to finding these richer, higher categorical structures include applications like constructing finer knot invariants, as well as proving positivity results and producing some fantastic mathematics. In this talk, I will focus on quantum groups. Their crystal bases or canonical bases exhibit the positivity and integrality that is a trademark feature of a decategorified structure. My launch point is the type A combinatorics of Young diagrams, which encode the representation theory of the symmetric group and also form a crystal—the crystal graph of the basic representation of \mathfrak{sl}_∞ . This is not a coincidence. The symmetric groups categorify the basic representation, with induction and restriction functors descending to raising and lowering operators. This phenomenon generalizes to all symmetrizable types, replacing the symmetric groups with cyclotomic Khovanov-Lauda-Rouquier (KLR) algebras.

Zoom: <https://reuna.zoom.us/j/81334559341>

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