Quadratic Forms Conference Patagonia 2013

List of speakers, Titles and Abstracts

Luis Arenas-Carmona

TITLE: *Some applications of the theory of representation of orders*

ABSTRACT: The theory of representation of orders in quaternion algebras helps us described certain family of quaternion quotient graphs studied by Serre in the seventies. This is done through an extension of the theory of genera and spinor genera to X-orders, where X is a projective curve defined over a finite field. The main tool used in the computation is a commutativity law between certain infinite matrices associated to the orders.

Karim Becher

TITLE: *Quadratic and cubic form invariants of certain algebras with involution*

ABSTRACT: In joint work with N. Grenier-Boley and J.-P. Tignol, we study algebras with involution whose maximal étale subextensions symmetric under the involution are of degree 4 over the center. This means that either the involution is orthogonal or unitary and the algebra is of degree 4, or the involution is symplectic and the algebra has degree 8. In this situation, we show the existence of a biquadratic symmetric subextension and that the decomposability of the algebra with involution is characterized by of the isotropy of a certain Pfister form. The treatment and the results are characteristic independent. To show the existence of biquadratic subextensions we use a certain cubic form. We further obtain a new proof for Rowen's theorem showing the existence of triquadratic subfields in central simple algebras of degree 8.

Siegfried Böecherer

TITLE: *Towards analytic extremality for maximal lattices with prime discriminant*

ABSTRACT: A few years ago G.Nebe and I showed that one can define analytic extremality for adjoints of maximal lattices L of level N, provided that the determinant of L is N^2 (and N odd). In this talk I discuss the case of squarefree determinants; this case is much more subtle: many representation numbers are zero by arithmetic reasons, furthermore the theory of modular forms of nebentypus is more complicated. For prime level we reduce the problem to an interesting property of level one modular forms mod p.

Juan Marcos Cerviño

TITLE: *Generalized Theta-Series*

ABSTRACT: In this talk I shall report on old and new results concerning lattice invariants (in particular in dimensions 4 and 16) – joint with G. Hein (Essen). These invariants, introduced via the heat kernel, can be naturally interpreted by classical invariant theory and also by a rather more general approach of

Ibukiyama. I address the question of completeness for these invariants and give some results, examples and evidence for a possible general statement.

Vladimir Chernousov

TITLE: *Lower bounds for essential dimensions of adjoint groups in characteristic 2*

ABSTRACT: We will show that for a simple adjoint group G defined over a field of characteristic 2 the essential dimension of G is at least $n+1$ where n is its rank. As an application we will show that the essential dimension of an orthogonal group $O(f)$ where f is a nondegenerate quadratic form over a field of characteristic 2 of dimension n is $m+1$ if $n=2m-1$ or $n=2m$.

Sungmun Cho

TITLE: *A uniform construction of smooth integral models and a recipe for computing local densities of any forms over any local fields*

ABSTRACT: In this talk we will explain a simple and uniform construction of smooth integral models associated to quadratic, (anti-) hermitian, (anti-) quaternionic hermitian lattices defined over an arbitrary non-Archimedean local field. As one major application, this construction gives a new, simple and effective recipe for computing local densities of the above lattices.

Renaud Coulangeon

TITLE: *Units in semisimple algebras over $\mathbb{Q}$ and Voronoï algorithm*

ABSTRACT: I will explain how the classical *Voronoï algorithm* can be used to perform computations with units in maximal orders of semisimple algebras. More specifically, I will describe first an algorithm which, given such a maximal order Λ, returns the conjugacy classes of maximal finite subgroups in its unit group $\Lambda^\times$. As time allows, I will also explain how a combination of the previous ideas with Bass-Serre theory of *graphs of groups* permits, in principle, to get a presentation (generators and relations) of $\Lambda^\times$.

Rainer Dietmann

TITLE: *On the representation of quadratic forms by quadratic forms*

ABSTRACT: It is a classical problem in Analytic Number Theory to obtain asymptotic formulas for the number of representations of positive integers by positive definite quadratic forms. One can generalise this question to the representation of positive definite quadratic forms B by positive definite quadratic forms A. Here one has the famous Siegel mass formula, which averages representation numbers over all forms in the genus of A, and Raghavan, using Siegel modular forms, obtained results for representations by individual forms A, given that the dimension of A is big enough in terms of the dimension of B, and providing that all successive minima of B are of comparable size. In this talk we give a survey on this problem and report on recent joint work with Michael Harvey, where we use an approach based on the circle method. This yields asymptotic formulas for representations by individual A without that assumption

on the minima of B, at the expense of needing bigger dimension of A in terms of B as opposed to Raghavan's result.

Andrew Earnest

TITLE: *Quadratic lattices with regularity properties*

ABSTRACT: Let F be a totally real algebraic number field with ring of integers $\mathcal{O}$. A definite quadratic $\mathcal{O}$-lattice L is said to be (strictly) k-regular for some positive integer k if L (primitively) represents all $\mathcal{O}$-lattices K of rank k that are (primitively) represented by its genus. In particular, any lattice of rank n having class number one is strictly k-regular for all $k \leq n$. In this talk, we will focus on the question of whether the number of nonisometric (strictly) k-regular lattices of rank n is finite, for various values of k and n. We will give an overview of known results of this type, and describe some new results in two specific cases: strictly 1-regular lattices of rank 4 over $\mathbb{Z}$, and $(n-1)$-regular lattices of rank n over arbitrary number fields F.

Lenny Fukshansky

TITLE: *Well-rounded lattices from algebraic constructions*

ABSTRACT: Well-rounded lattices are vital in extremal lattice theory, since the classical optimization problems can usually be reduced to them. On the other hand, many of the important constructions of Euclidean lattices with good properties come from different algebraic settings. This prompts a natural question: which of the lattices coming from algebraic constructions are well-rounded? We consider three such well known algebraic constructions: ideal lattices from number fields, cyclic lattices from quotient polynomial rings, and function field lattices from curves over finite fields. In each of these cases, we provide a partial answer to the above question and propose directions for future research.

Skip Garibaldi

TITLE: *Simple algebraic groups and polynomial invariants*

ABSTRACT: The classical "linear preserver problem" asks: Given a polynomial in finitely many variables, what is the group of invertible linear transformations that preserve it? This problem has been solved for many interesting polynomials, usually by means that are special to the particular polynomial under consideration. We turn this problem on its head by starting with a polynomial that is preserved by a simple algebraic group and observe that the full preserver can be described by a general theorem. As a special case, we address a 125+ year old problem. These results are joint work with Bob Guralnick.

Stefan Gille

TITLE: *On Brauer groups of affine quadrics and some other varieties*

ABSTRACT: We give a method to compute the n-torsion of the Brauer group of certain varieties which include affine quadrics or some products of algebraic groups with tori if the characteristic of the base field does not divide the integer

n. We show also that given a field k of characteristic $p > 0$ and a finitely generated k-algebra R which is geometrically integral of dimension ≥ 2 then the natural map from the Brauer group of k to the one of R is never surjective.

Paweł Gładki

TITLE: *Counting Witts*

ABSTRACT: We shall investigate Witt equivalence of function fields of rational conic sections and try to count as many non-equivalent Witt classes of such fields as possible. The results to be presented rely heavily on a number of theorems published already in the 1930s and provide new insight into the history of the algebraic theory of quadratic forms. This is joint work with Murray Marshall.

David Grimm

TITLE: *Sums of squares of rational functions on real algebraic or arithmetic surfaces*

ABSTRACT: We discuss some quantitative results on sums of squares in function fields of real surfaces. In particular, we show that the Pythagoras number of the function field of an algebraic surface over $\mathbb{R}$ is either three or four, while that of an arithmetic surface over $\mathbb{R}[\![t]\!]$ is either two or three. In the latter case, we measure by how far the function fields fail to have Pythagoras number two, that is, we bound the order of sums of squares modulo sums of two squares in terms of the genus of the function field.

Max Karoubi

TITLE: *Witt groups of real algebraic varieties*

ABSTRACT: Let V be an algebraic real variety. Our purpose is to compute the Witt group of V as a function of topological invariants. Our starting point is Brumfiel's theorem: the comparison map $r : W(V) \to KO(V')$ between the algebraic Witt group and its topological analog $KO(V')$, where V' is the set of real points, is an isomorphism modulo 2 primary torsion. A better invariant than $KO(V')$ is the Witt group $WR(X)$ of the category of real bundles (in the sense of Atiyah) on the space X of complex points. The group $WR(X)$ is a topological invariant of the real space X: Brumfiel's map r factors as $W(V) \to WR(X) \to KO(V')$. The second map is an isomorphism modulo 2-primary torsion and therefore $WR(X)$ plays the same role as $KO(V')$ in Brumfiel's theorem.

In this lecture we give an upper bound for the exponent of the groups $\mathrm{Ker}(c)$ and $\mathrm{Coker}(c)$ where c is the first map $W(V) \to WR(X)$ according to the dimension of V: This result sharpens Brumfiel's theorem. For the proofs, we use in an essential way the computation of the K-theory of real varieties due to Karoubi and Weibel, Williams' conjecture describing Hermitian K-theory as a homotopy fixed point set, a result due to Berrick, Karoubi, Schlichting and Ostvaer, and finally Bott periodicity in an algebraic setting. An interesting computation is the group $WR(X)$ when V is a real curve, where we recover some results of Knebusch by topological methods. In this case, the topological invariants are

the genus and the number of real connected components. This is joint work with Marco Schlichting and Charles Weibel.

Nikita Karpenko

TITLE: *Minimal canonical dimensions of quadratic forms*

ABSTRACT: The canonical dimension of a smooth complete variety is the minimal dimension of the images of its rational endomorphisms. The i-th canonical dimension of a non-degenerate quadratic form is the canonical dimension of its i-th orthogonal grassmannian. The maximum of a canonical dimension of quadratic forms of a fixed dimension is the dimension of the corresponding grassmannian. This talk is about the minima of the canonical dimensions of an anisotropic quadratic form.

Abhinav Kumar

TITLE: *Lattices and K3 surfaces*

ABSTRACT: The study of K3 surfaces is closely connected with the study of indefinite lattices. More precisely, to a K3 surface we can associate its Neron-Severi group, an even Lorentzian lattice which sits inside the second cohomology lattice $E_8^2 + U^3$. I will survey how this connection informs the geometry and moduli theory of K3 surfaces, and gives us some clues about the arithmetic. In addition to describing the classical story, I will also mention several recent advances and open questions in this field.

Ahmed Laghribi

TITLE: *On the descent problem for bilinear forms in characteristic* 2.

ABSTRACT: Let F be a field of characteristic 2, $K = F(q)$ be the function field of the projective quadric given by an F-quadratic form q, and B a bilinear form over K such that $B + I^n K$ belongs to the image of the natural homomorphism $W(F)/I^n F \longrightarrow W(K)/I^n K$ for some integer n. Our aim is to give conditions on q and n for which B is definable over F, i.e. there exists a bilinear form C over F such that B is isometric to C_K. We will do this in the following two cases: (i) q is not totally singular and B is arbitrary; (ii) q is totally singular and B is of dimension less than or equal to 4. As an application, we give a complete description of the splitting patterns of bilinear forms of dimension ≤ 9.

Emilio Lauret

TITLE: *An asymptotic formula for representations of integers by indefinite hermitian forms.*

ABSTRACT: By applying a lattice point theorem on n-dimensional real (and complex) hyperbolic spaces, we gave an asymptotic formula with an error term for the number of integral representations of a negative integer by an indefinite quadratic (and hermitian) forms of signature $(n, 1)$. The error term depends on the first nonzero eigenvalue of the Laplace-Beltrami operator in certain hyperbolic manifolds. We also study the behavior of the error term with experimental

computations, obtaining evidences on the existence of exceptional eigenvalues in certain complex hyperbolic manifolds.

David Leep

TITLE: *A survey of results on the u-invariant of a rational function field*

ABSTRACT: The u-invariant of a field is the maximal dimension of an anisotropic quadratic form defined over the field. This talk will survey the known results for computing the u-invariant of a rational function field. I will mention classical examples, recent results on the u-invariants of p-adic function fields, a generalization of a counter-example due to Colliot-Thélène and Madore, and other more general, but partial, results for arbitrary rational function fields.

Benjamin Linowitz

TITLE: *The arithmetic of quaternion orders and isospectral hyperbolic surfaces*

ABSTRACT: A well-known construction associates to an order in a quaternion algebra (defined over a totally real number field) a hyperbolic surface. In 1980 Vigneras used this construction in order to prove the existence of hyperbolic surfaces which were isospectral (have the same spectrum with respect to the Laplace-Beltrami operator) but not isometric. Key to Vigneras' method was a characterization of the values contained in the spectrum of an arithmetic manifold as embedding numbers of certain rank two commutative orders into quaternion orders. In this talk we will review the embedding theory of quaternion orders and show how the notion of "selectivity" may be used to construct isospectral hyperbolic surfaces of extremely small volume. We will further show that our examples have minimal volume amongst all isospectral hyperbolic surfaces arising from congruence arithmetic Fuchsian groups. This is joint work with Peter Doyle and John Voight.

Alexander Merkurjev

TITLE: *Cohomological invariants of algebraic groups*

ABSTRACT: The notion of a cohomological invariant of an algebraic group was introduced by J-P. Serre. A survey of old and new results will be given in the talk. In particular, semi-decomposable degree 3 invariants will be discussed. This is a joint work with A. Neshitov and K. Zainoulline.

Byeong-Kweon Oh

TITLE: *Binary quadratic forms representing same multiples of a prime*

ABSTRACT: In 1938, Delone proved that (x^2+3y^2, x^2+xy+y^2) is the unique pair of non-isometric positive definite integral binary forms representing the same set of integers. In this talk, we describe the result of determining all pairs of positive definite binary forms representing the same set of multiples of p, for a given prime p.

Parimala

TITLE: *u-invariant and uniform boundedness for the Brauer group*

ABSTRACT: Saltman's theorem brings out the equivalence of the finiteness of the u-invariant of a field and finiteness of the symbol length in Galois cohomology. Using a recent result of Krashen, one can show that the finiteness of the u-invariant follows from 'uniform boundedness' for the 2-torsion in the Brauer group. Function fields of curves over complete discrete valued fields with residue fields of characteristic 2 and finite p-rank have uniform boundedness for the Brauer group. This leads to the finiteness of the u-invariant of such fields. (Joint work with Suresh.)

Jeremy Rouse

TITLE: *Quadratic forms representing all odd positive integers*

ABSTRACT: We consider the problem of classifying all positive-definite integer-valued quadratic forms that represent all positive odd integers. Kaplansky considered this problem for ternary forms, giving a list of 23 candidates, and proving that 19 of those represent all positive odds. (Jagy later dealt with a 20th candidate.) Assuming that the remaining three forms represent all positive odds, we prove that an arbitrary, positive-definite quadratic form represents all positive odds if and only if it represents the odd numbers from 1 up to 451. This result is analogous to Bhargava and Hanke's celebrated 290-theorem. In addition, we prove that these three remaining ternaries represent all positive odd integers, assuming the Generalized Riemann Hypothesis.

Alexander Sivatski

TITLE: *On zeros of certain quadratic forms over the rational function field*

ABSTRACT: Let k be a field of characteristic different from 2, $d \in k$. Let further φ and ψ be quadratic forms over k, $\dim \varphi + \dim \psi = n$. Suppose that the form $\Phi = \varphi \perp (t^2 d)\psi$ is isotropic over the rational function field $k(t)$. We prove that the anisotropic part of the form Φ has the same structure, i.e. $\Phi_{an}\varphi_1 \perp (t^2 d)\psi_1$ for some forms φ_1 and ψ_1 over k. As a consequence we show that there exist polynomials $p_1(t), \dots, p_n(t) \in k[t]$ not simultaneously zero such that $\Phi(p_1(t), \dots, p_n(t)) = 0$, and $\deg p_i \leq \frac{\dim \Phi_{an}}{i_0(\Phi)} + 1$ for each i, where $i_0(\Phi)$ is the Witt index of Φ (note that because of quadratic coefficients the algorithm in the Cassels-Pfister theorem does not improve a zero of Φ, hence it cannot be used to obtain an upper bound for $\deg p_i$). We also consider certain particular cases where the above inequality can be improved, and a few related open questions.

Thomas Unger

TITLE: *Signatures of hermitian forms, Knebusch trace formula and "prime ideals" of Witt groups*

ABSTRACT: I will survey my recent work with V. Astier on signatures of hermitian forms over central simple algebras with involution with respect to orderings on the base field.

Alexander Vishik

TITLE: *A new homotopic approach to the classification of quadratic forms and other torsors*

ABSTRACT: I will describe a new approach to the classification of torsors. Here torsors are interpreted as homotopy group elements of the classifying space of a group. Working in the homotopy category $H(k)$ of Morel-Voevodsky we produce a new type of invariants of torsors of a very general nature. In the case of quadratic forms, these invariants are richer and more subtle than what was known previously, which lets us move deeper into the classification. We establish the explicit relation of our invariants to the "classical" ones which permits to incorporate the well-known invariants into a much larger picture, and relate previously isolated fields. This is joint work with Alexander Smirnov.

Fei Xu

TITLE: *Strong approximation with Brauer-Manin obstruction for certain algebraic varieties*

ABSTRACT: One of the immediate application of the strong approximation with Brauer-Manin obstruction is to determine the existence of integral points. In this talk, I will explain the recent progress for strong approximation with Brauer-Manin obstruction for certain class of algebraic varieties. Some interesting examples will be given.

Quadratic Forms Conference

Patagonia 2013

	Monday 16	Tuesday 17	W. 18	Thursday 19	Friday 20
9:15-10:15	R. Parimala	R. Dietmann		D. Leep	S. Garibaldi
10:20-11:00	R. Coulangeon	T. Unger		B. Linowitz	B.-K. Oh
11:05-11:35	**C. BREAK**	**C. BREAK**		**C. BREAK**	**C. BREAK**
11:35-12:15	S. Gille	L. Arenas		S. Boecherer	P. Gladki
12:20-13:00	S. Cho	A. Laghribi		V. Chernousov	L. Fukshansky
13:15-15:00	**LUNCH**	**LUNCH**		**LUNCH**	**LUNCH**
15:00-16:00	A. Merkurjev	F. Xu		A. Vishik	A. Kumar
16:10-16:50	J. Cervino	K. Becher		E. Lauret	N. Karpenko
16:55-17:25	**C. BREAK**	**C. BREAK**		**C. BREAK**	**C. BREAK**
17:25-18:05	D. Grimm	J. Rouse		A. Sivatski	A. Earnest
18:10-18:50		M. Karoubi			