# Local strong decay estimates in some nonlinear dispersive equations with supercritical, modified scattering

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### Abstract

This mini-course is intended to discuss some new results obtained in collaboration with M. Kowalczyk (U. Chile) and Y. Martel (E. Polytechnique, France) on scattering and decay estimates in some well-known nonlinear dispersive equations with critical or supercritical scattering behavior, and where standard dispersive estimates are not available yet.

In a first lecture, I will introduce the main topics and problems, explain basic linear decay estimates and discuss the current literature in the topic.

In the second lecture, I will show that for a large class of nonlinear wave equations with odd nonlinearities, any globally defined odd solution which is small in the energy space decays to zero in the local energy norm. In particular, this result shows nonexistence of small, odd *breathers* for some classical nonlinear Klein-Gordon equations.

Finally, in a last lecture I will consider a classical equation known as the $\phi^4$ model in one space dimension. The *kink* is an explicit, stable stationary solution of this model. In this lecture I will show "asymptotic stability" of the kink for odd perturbations in the energy space.

The proofs that we will discuss are based on Virial-type estimates partly inspired from previous works of Martel-Merle on asymptotic stability of solitons for the generalized Korteweg-de Vries equations, adapted to additional difficulties in the case of general Klein-Gordon equations with potential, that I will try to discuss in some detail. (See the bibliography below and references therein for a more complete literature.)

**Tentative schedule:**

1. *Lecture 1:* Introduction. Linear dynamics and known decay results in some classical nonlinear dispersive equations.
2. *Lecture 2:* Dynamics and decay around zero backgrounds: nonexistence of breathers in nonlinear wave equations.
3. *Lecture 3:* Dynamics in non-zero backgrounds: kink dynamics in the $\phi^4$ model.

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# References

[1] M. Kowalczyk, Y. Martel and C. Muñoz, *Kink dynamics in the $\phi^4$ model: asymptotic stability for odd perturbations in the energy space*, arXiv:1506.07420, JAMS 30 (2017), 769–798.

[2] M. Kowalczyk, Y. Martel and C. Muñoz, *Nonexistence of small, odd breathers for a class of nonlinear wave equations*, arXiv:1607.06421, L. Math. Phys. May 2017, Vol. 107, Issue 5, pp. 921–931.