Programación Sesión S05: Matemática Discreta

Jueves 02 de Noviembre: 15.40 - 17.10

- 15.40 - 16.10 **José A. Soto**, Universidad de Chile
- 16.10 - 16.40 **Anahí Gajardo**, Universidad de Concepción
- 16.40 - 17.10 **Pablo Pérez-Lantero**, Universidad de Santiago de Chile

Viernes 03 de Noviembre: 11.10 - 13.00

- 11.10 - 11.40 **José Zamora**, Universidad Andrés Bello
- 11.40 - 12.10 **Martín Matamala**, Universidad de Chile
- 12.10 - 12.35 **Matías Pavez Signé**, Universidad de Chile
- 12.35 - 13.00 **Guido Besomi**, Universidad de Chile

Viernes 03 de Noviembre: 15.40 - 17.40

- 15.40 - 16.10 **Sebastián Bustamante**, Universidad de Chile
- 16.10 - 16.40 **Andreas Wiese**, Universidad de Chile
- 16.40 - 17.10 **Daniel A. Quiroz**, Universidad de Chile
- 17.10 - 17.40 **Andrea Jiménez**, Universidad de Valparaíso

The ordinal secretary problem on graphs (and matroids)

José A. Soto *

Abstract

An edge-weighted graph is presented edge by edge in uniform random order to an algorithm that wants to construct a large weight forest. When an edge appears, the algorithm must decide whether to take it or not, without creating any cycle. But there is a catch: the algorithm cannot see the weights, it can only compare pairs of revealed elements. Can the algorithm output a forest whose weight is large compare to the optimum one?

If we replace "finding a forest in a graph" by "finding an independent set on a given matroid" we obtain the description of the ordinal matroid secretary problem (MSP).

In this talk, I will present a technique to design, for certain matroids, algorithms with the following property: every element of the global optimum appears in the output solution with constant probability. Our technique allows us to improve the current best competitive guarantees of the MSP on most of the studied matroid, including the graphical case above for which we achieve a 4-competitive factor.

This talk is based on recent work with Abner Turkieltaub and Victor Verdugo [1].

References

[1] José A. Soto, Abner Turkieltaub and Victor Verdugo. *Strong Algorithms for the Ordinal Matroid Secretary Problem.* To appear in the Twenty-Eighth Annual ACM-SIAM Symposium on Discrete Algorithms (SODA2018)

*Departamento de Ingeniería Matemática, Universidad de Chile. e-mail: jsoto@dim.uchile.cl

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AUTÓMATAS CELULARES PRE-EXPANSIVOS

ANAHÍ GAJARDO *

Abstract

La expansividad es una forma de sensitividad muy fuerte, afirma que para cualquier punto de partida, cualquier perturbación se alejará del punto inicial en algún momento. En el contexto de autómatas celulares, una perturbación pequeña es una que afecta a celdas lejanas del centro del espacio, pero podra afectar a una cantidad infinita de celdas sin por esto dejar de ser pequeña. En esta charla haremos el estado del arte sobre una noción ligeramente más débil, la *pre-expansividad*. Esta noción, introducida en [1], es más débil pues solo exige que las perturbaciones sobre un conjunto finito de celdas lleguen a afectar todo el espacio. Se encuentra así justo entremedio de la expansividad y la sensitividad a condiciones iniciales.

Estudiamos el caso unidimensional, caso en el cual se prueba que la pre-expansividad es una nocin fuerte de sensitividad puesto que implica la sensitividad direccional en todas las direcciones posibles. Mostramos además varios ejemplos de autómatas pre-expansivos reversibles, que no son expansivos pues la expansividad es imposible en el contexto reversible.

Estudiamos también la pre-expansividad en dimensiones superiores. Allí lamentablemente, basándonos en [2], demostramos un resultado negativo que dice que no existen autómatas pre-expansivos lineales en dimensión mayor o igual a 2. Esto indica que de existir un autómata pre-expansivo, este debe ser no lineal, y el problema queda abierto. Cabe destacar, que en este contexto no existen tampoco autómatas expansivos [3].

Finalmente, estudiamos una noción més débil: la k-expansividad, que dice que las perturbaciones en exactamente k celdas afectan toda la evolución de un punto. Sorprendentemente, sí hay ejemplos de autómatas bi-dimensionales que son 1-expansivos y 3-expansivos, pero no 2-expansivos.

Trabajo realizado en conjunto con:
Vincent Nesme, Lycée Georges Brassens, France.
Guillaume Theyssier[1], Institut de Mathématiques de Marseille - CMM - CNRS, Aix Marseille Universié, Marseille, France.

*Departamento de Ingeniería Matemática y Centro de Investigación en Ingeniería Matemática, Universidad de Concepción. Parcialmente financiado por FONDECYT #1140684 y CONYCYT- BASAL PFB03 e-mail: `anahi@ing-mat.udec.cl`

[1] Parcialmente financiado por CONYCYT- Basal PFB03, e-mail: `guillaume.theyssier@cnrs.fr`

References

[1] A. Gajardo, V. Nesme, G. Theyssier *Pre-Expansivity in Cellular Automata*, CoRR abs/1603.07215, 2017.

[2] J. Gtschow, V. Nesme, R. F. Werner. *The fractal structure of cellular automata on abelian groups*, en: Proceedings of AUTOMATA 2010, pp 5574, INRIA Nancy Grand Est, 2010.

[3] M. Pivato. *Positive expansiveness versus network dimension in symbolic dynamical systems*, Theoretical Computer Science, 412(30):3838-3855, 2011.

Maximum Box Problem on Stochastic Points

Pablo Pérez-Lantero *

Abstract

Given a finite set of weighted points in $\mathbb{R}^d$ (where there can be negative weights), the maximum box problem asks for an axis-aligned rectangle (i.e., box) such that the sum of the weights of the points that it contains is maximized. We consider that each point of the input has a probability of being present in the final random point set, and these events are mutually independent; then, the total weight of a maximum box is a random variable. We aim to compute both the probability that this variable is at least a given parameter, and its expectation. We show that even in $d = 1$ these computations are #P-hard, and give pseudo polynomial-time algorithms in the case where the weights are integers in a bounded interval. For $d = 2$, we consider that each point is colored red or blue, where red points have weight $+1$ and blue points weight $-\infty$. The random variable is the maximum number of red points that can be covered with a box not containing any blue point. We prove that the above two computations are also #P-hard, and give a polynomial-time algorithm for computing the probability that there is a box containing exactly two red points, no blue point, and a given point of the plane.

This is a joint work with L. E. Caraballo, C. Seara, and I. Ventura.

*Departamento de Matemática y Ciencia de la Computación, Universidad de Santiago de Chile (USACH), e-mail: `pablo.perez.l@usach.cl`

Lines in Graphs

José Zamora *

Abstract

A well-known combinatorial theorem says that a set of n non-collinear points in the plane determines at least n distinct lines. In [1] Chen and Chvtal conjectured that this theorem extends to metric spaces, with an appropriated definition of line.

One special case is when the metric space is induced by a graph. In this case, the points are the vertices of the graph and the distance between two points is given by the length of the its shortest path.

In this talk, I will present the principal results in this topic. I will also propose a hierarchy of graphs with respect to its number of lines as a plan to study the conjecture.

References

[1] X. Chen and V. Chvátal, Problems related to a de Bruijn - Erdős theorem, *Discrete Applied Mathematics* **156** (2008), 2101–2108.

*Departamento de Matemáticas , Universidad Andres Bello , e-mail: `josezamora@unab.cl`

Matching in colored graphs

Martín Matamala *

Abstract

A matching of a graph $G = (V, E)$ is a set of edges M such that for each vertex v the set $E(v) = \{uv \in E, u \in V\}$ contains at most one element of M. Let $\alpha : V \to \mathbb{N}$. A α-matching is a function $\beta : E \to \mathbb{N}$ such that for each vertex $v \in V$, the set $\{uv \in E(v) : \beta(uv) \in \{\alpha(u), \alpha(v)\}\}$ contains at most one element. When $\alpha(u) = \alpha(v)$ for all $uv \in E$, an α-matching can be identified with the set $\{uv \in E : \beta(uv) = \alpha(u)\}$ which is a matching of G.

In this work we show that Tutte's Theorem for perfect matching has an extension into the context of α-matching.

*DIM-CMM, Universidad de Chile, e-mail: `mmatamal@dim.uchile.cl`

Embedding de árboles con grado máximo acotado

Parte 1

Matías Pavez Signé *

Abstract

Sea T_k cualquier árbol en k aristas y grado máximo acotado por Δ, estudiaremos qué tipo de condiciones debe tener un grafo G de modo de contener una copia de T_k como subgrafo. Nuestro resultado principal establece que si G es un grafo con grado mínimo al menos $(1+\delta)\frac{k}{2}$ y grado máximo al menos $(1+\delta)2k$, entonces $T_k \subseteq G$. Además, los métodos utilizados servirán para dar respuestas parciales a dos conjeturas referentes a *embedding* de árboles: la conjetura $\frac{2k}{3}$ y la conjetura de Erdős-Sós.

References

[1] Havet, F., Reed, B., Stein, M., Wood, D. (2016) A Variant of the Erdős-Sós Conjecture. *Manuscript.*

[2] Komlós, J., Sárközy, G. N. and Szemerédi, E. (1995) Proof of a packing conjecture of Bollobás. *Combin. Probab. Comput.* **4** 241-255.

[3] Komlós, J., Sárközy, G. N. and Szemerédi, E. (2001) Spanning Trees in Dense Graphs. *Combin. Probab. Comput.* **10** 397-416

*Departamento de Ingeniería Matemática, Universidad de Chile , e-mail: mpavez@dim.uchile.cl

Embedding de árboles con grado máximo acotado

Parte 2

Guido Besomi *

Abstract

Sea T_k cualquier árbol en k aristas y grado máximo acotado por Δ, estudiaremos qué tipo de condiciones debe tener un grafo G de modo de contener una copia de T_k como subgrafo. Nuestro resultado principal establece que si G es un grafo con grado mínimo al menos $(1+\delta)\frac{k}{2}$ y grado máximo al menos $(1+\delta)2k$, entonces $T_k \subseteq G$. Además, los métodos utilizados servirán para dar respuestas parciales a dos conjeturas referentes a *embedding* de árboles: la conjetura $\frac{2k}{3}$ y la conjetura de Erdős-Sós.

References

[1] Havet, F., Reed, B., Stein, M., Wood, D. (2016) A Variant of the Erdős-Sós Conjecture. *Manuscript.*

[2] Komlós, J., Sárközy, G. N. and Szemerédi, E. (1995) Proof of a packing conjecture of Bollobás. *Combin. Probab. Comput.* **4** 241-255.

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*Departamento de Ingeniería Matemática, Universidad de Chile , e-mail: gbesomi@dim.uchile.cl

Monochromatic ℓ-cycle partitions of $\mathcal{K}_n^{(k)}$

Sebastián Bustamante *

Abstract

Given an edge-colouring of a graph or hypergraph $\mathcal{K}$, the problem of partitioning the vertices of $\mathcal{K}$ into the smallest number of monochromatic cycles has received much attention. Central to this area has been an old conjecture of Lehel [1] stating that two monochromatic disjoint cycles in different colours are sufficient to cover the vertex set of any 2-edge-colouring of the complete graph K_n. This conjecture was confirmed 31 years later by Bessy and Thomassé in [2]. In the hypergraph setting, an ℓ-cycle is a set of cyclically ordered edges such that consecutive edges intersect in exactly ℓ vertices and non consecutive edges are disjoint. We show that for sufficiently large n and every $0 < \ell \leq k/2$, two monochromatic ℓ-cycles in different colours are sufficient to partitition all but at most a constant $c_{k,\ell}$ of vertices of any 2-edge-colouring of the complete k-uniform hypergraph $\mathcal{K}_n^{(k)}$. We also discuss several generalisations and open problems.

This is joint work Maya Stein.

References

[1] Jacqueline Ayel. *Sur l'existence de deux cycles supplémentaires unicolores, disjoints et de couleurs différentes dans un graphe complet bicolore.* PhD thesis, Université Joseph-Fourier-Grenoble I, 1979.

[2] Stéphane Bessy and Stéphan Thomassé. Partitioning a graph into a cycle and an anticycle, a proof of Lehel's conjecture. *Journal of Combinatorial Theory, Series B*, 100:176–180, 2010.

*Departamento de Ingeniería Matemática, Universidad de Chile, e-mail: `sbustamante@dim.uchile.cl`

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Parameterized $(1+\epsilon)$-approximation algorithms for packing problems

Fabrizio Grandoni* Stefan Kratsch† Andreas Wiese‡

Abstract

Approximation algorithms and parameterized algorithms are two well-established ways to deal with NP-hard problems. In the first case, the goal is to compute in polynomial time a solution of cost close to the optimum. In the second case, the goal is to compute an optimal solution in (FPT) time $f(k)n^{O(1)}$, where k is some parameter of the input. The recent area of parameterized approximation seeks to combine the two approaches by, e.g., aiming for $(1+\epsilon)$-approximations in $f(k,\epsilon)n^{g(\epsilon)}$ time.

We present such algorithms for three important packing problems: for the Maximum Independent Set of Rectangles problem, for the Unsplittable Flow on a Path problem, and for the Two-Dimensional Knapsack problem with rotations. All three problems are W[1]-hard and hence we do not expect to find an FPT-algorithm for them. Also, it seems very difficult to construct a PTAS for them which motivates studying parameterized $(1+\epsilon)$-approximations.

*IDSIA, USI-SUPSI, Switzerland, `fabrizio@idsia.ch`.
†University of Bonn, Germany, `kratsch@cs.uni-bonn.de`
‡Universidad de Chile, `awiese@dii.uchile.cl`

Generalised colouring numbers of planar graphs

Daniel A. Quiroz *

Abstract

The generalised colouring numbers $col_r(G)$ and $wcol_r(G)$ were introduced by Kierstead and Yang as a generalisation of the colouring number, and have found important theoretical and algorithmic applications.

We will discuss an improvement on the known upper bounds to these numbers for graphs excluding a complete graph as a minor, from the exponential bounds of Grohe *et al.* to a linear bound on $col_r(G)$ and an polynomial bound on $wcol_r(G)$. We will look at some of the main techniques used to obtain these new bounds, with an emphasis on the results for planar graphs.

This is joint work with Jan van den Heuvel, Patrice Ossona de Mendez, Roman Rabinovich and Sebastian Siebertz.

*Centro de Modelamiento Matemático, Universidad de Chile, Santiago, Chile. e-mail: dquiroz@dim.uchile.cl

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Deciding whether a Grid is a Topological Subgraph of a Planar Graph is NP-Complete

Andrea Jiménez *

Abstract

The Topological Subgraph Containment (TSC) Problem is to determine for two given graphs G and H, whether H is a topological subgraph of G. On the one hand, an algorithmic implication of the famous Graph Minor Theorem of Robertson and Seymour is that, when H is fixed, the TSC Problem can be solved in time polynomial in the order of the input graph G. On the other hand, when H is part of the input, the problem is NP-complete even when restricted to planar input graphs. Grids are fundamental structures in graph algorithms due to its close relation to the treewidth.

In this work, we study the computational complexity of the following decision problem: given a positive integer k and a planar graph G, is the $k \times k$ grid a topological subgraph of G? We prove that this problem is NP-complete, even when restricted to planar graphs of maximum degree six, via a reduction from the Planar Monotone 3-SAT Problem. This is joint work with Tina Janne Schmidt.

*CIMFAV, Universidad de Valparaíso, e-mail: andrea.jimenez@uv.cl