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Around the convergence of multiple ergodic averages

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Abstract

A milestone in classical ergodic theory is the proof given by Furstenberg of the existence of arbitrarily long arithmetic progressions in a subset of the integers with positive density (the celebrated Szemerédi's Theorem), that opened a fruitful connection between ergodic theory, number theory and additivie combinatorics. His proof introduced the *multiple ergodic averages* as the main object of study, and whose limiting behaviour has been studied by several authors.

In this talk I will review this topic in ergodic theory, focusing on the problem of convergence of the multiple ergodic averages. I will survey the known results about L^2 convergence and talk about the new ones in the pointwise case.

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