PRYM VARIETIES AND PRYM MAP
COMPLEMENTS

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1st Tutorial

(1) Let $A = V/\Lambda$ be a complex torus. Choose $\{e_i\}$ basis of V and $\lambda_1, \dots, \lambda_{2g}$ of Λ and Π the corresponding period matrix, i.e. w.r.t. these basis $A = \mathbb{C}^g/\Pi\mathbb{Z}^{2g}$. Show that A is an abelian variety if and only if there is a non degenerated alternating form $Z \in M_{2g}(\mathbb{Z})$ such that (i) $\Pi Z^{-1\,t}\Pi = 0$ and (ii) $i\Pi Z^{-1\,t}\overline{\Pi} > 0$. The conditions (i) and (ii) are called Riemann Relations. (If E is the alternating form on Λ defining the polarization, Z is the matrix of E on the basis $\{\lambda_i\}$). ([1, Thm. 4.2.1])

(2) Show the universal property of the Jacobian. ([1, Thm. 11.4.1])

(3) Let $f : \widetilde{C} \to C$ be an étale double covering and σ the corresponding involution on $\widetilde{C}$. Show that $\operatorname{Ker}\operatorname{Nm}_f : J\widetilde{C} \to JC$ consists of 2 irreducible components that can be described as $P^0 := (1-\sigma)\operatorname{Pic}^0(\widetilde{C})$ and $P^1 := (1-\sigma)\operatorname{Pic}^1(\widetilde{C})$.

(4) Let $f : \widetilde{C} \to C$ a finite covering between smooth curves. Show that the pullback map $f^* : JC \to J\widetilde{C}$ is not injective if and only if f factorizes via a cyclic étale covering $\tilde{f}$ of degree ≥ 2. ([1, Prop. 11.4.3])

2nd Tutorial

(1) Explain why the cotangent bundle to the moduli space of marked curves $\mathcal{M}_{g,r}$ at the point $[C, B]$ with $B = p_1 + \cdots + p_r$ is $H^0(C, \omega_C(B))$ (Ignacio is willing to do this part).

(2) Explain why the cotangent bundle to the moduli space of polarized abelian varieties $\mathcal{A}_g^D$ at the point (A, L) is isomorphic to $Sym^2(T_0^*A)$.

3rd Tutorial

This tutorial can devote to the Recillas trigonal construction and Donagi's tetragonal construction. The goal is to proof (or at least give an idea) that tetragonal related coverings have the same Prym variety.

References

[1] Ch. Birkenhake, H. Lange, *Complex abelian varieties*, Second edition. Grundlehren der Mathematischen Wissenschaften, 302. Springer-Verlag (2004).

[2] R. Donagi, *The tetragonal construction*, Bull. Amer. Math.Soc. (N.S.) **4** (1981), 181–185.

[3] R. Donagi, *The fibers of the Prym map*, Curves, Jacobians, and abelian varieties (Amherst), Contemp. Math., 136, 55–125.

[4] D. Mumford, Prym varieties I, Contributions to analysis (a collection of papers dedicated to Lipman Bers), 325–350, Academic Press, New York, 1974.

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